ROLE OF TANGENTIAL STRESSES IN THE MEASUREMENT OF NORMAL STRESSES BY AN INTERFERENCE METHOD

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The negligibly small influence of tangential stresses in an optically sensitive material as compared to the influence of the normal stresses on the displacement of interference fringes is shown for the use of a definite scheme.

The effects of normal stresses in fluids are investigated in [1] by using the interference of large path differences and the property of epoxy resin to change the index of refraction under a load. The shift of the interference fringes was an indicator of the change in normal stresses. The diagram to obtain the measuring system for the fringes is represented in Fig. 1.

Results of investigating the influence of the temperature factor, which is always substantial for interference measurement, are elucidated in [2].

To verify the correctness of the normal stress measurements, the influence of the tangential stresses acting on the epoxy resin layer from the fluid being subjected to a shear force and also the changing refractive index of the resin must be taken into account. And although the contribution to the shift of the interference fringes from the tangential stresses should not be substantial since the direct optical coefficient of the stress exceeds the transverse factor by an order of magnitude for amorphous glassy materials [3], a theoretical estimate and an experimental confirmation are necessary.

The role of the tangential stresses can be estimated by starting from the general theory of artificial anisotropy. Here the refractive index of the material is related to the dielectric permittivity [4] $n_i^2 = \varepsilon_i$. The whole theory of photoelasticity is constructed under the assumption of coincidence of the principal axes of the stress tensor and the principal dielectric axes. As is known from elasticity theory [5], the coordinate axes can always be selected so that the tensor would have diagonal form. The presence of tangential stresses means that the tensor is not diagonal, and therefore, the problem of the influence of the tangential stresses can be reduced to determining the influence of rotation of the dielectric ellipsoid on the change in the dielectric properties along a definite direction.

According to the theory of artificial anisotropy for isotropic preloading of the material, the ellipsoid of the wave normals has spherical form. The anisotropy occurring under a load transforms the sphere into an ellipsoid. The differences between the new and old coefficients of the equation for the ellipsoid (sphere) are linear functions of the stress tensor components. Taking account of requirements on the proportionality factors [6], we obtain expressions for these differences

$$
\begin{aligned}
a_{xx} - \frac{1}{\varepsilon} &= \alpha p_{xx} + \beta p_{yy} + \beta p_{zz},\\
a_{yy} - \frac{1}{\varepsilon} &= \beta p_{xx} + \alpha p_{yy} + \beta p_{zz},\\
a_{zz} - \frac{1}{\varepsilon} &= \beta p_{xx} + \beta p_{yy} + \alpha p_{zz},\\
a_{yz} = \gamma p_{yz};\quad a_{zx} &= \gamma p_{zx};\quad a_{xy} = \gamma p_{xy},
\end{aligned}
\tag{1}
$$

where α, β, γ are the proportionality factors, and ε is the dielectric constant in the unloaded state.

In the two-dimensional case of interest to us, if the light proceeds along the z axis, the boundary conditions take the form $p_{zz}|_{z=0} = p_{zx}|_{z=0}$; $p_{xx}|_{z=0} = 0$ and system (1) reduces to

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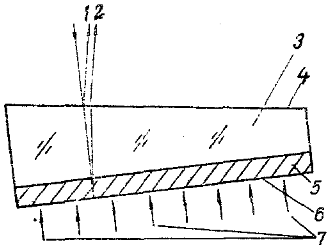

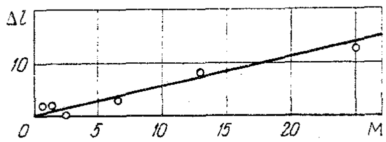

Fig. 1 Fig. 2

Fig. 1. Diagram to obtain the measuring system of the interference fringes: 1, 2) integrating light beams; 3) wedge-shaped glass preventing the deflection of the resin layer; 4) semitransparent coating; 5) optically sensitive epoxy resin layer; 6) mirror coating; 7) pressure distribution on the resin under investigation.

Fig. 2. Influence of tangential stresses on the shift in the interference fringes, Δl, μm.

$$a_{xx} = \frac{1}{\varepsilon} + \beta p_{zz}; \quad a_{zz} = \frac{1}{\varepsilon} + \alpha p_{zz}; \quad a_{zx} = \frac{(\alpha-\beta)}{2} p_{zx}, \tag{2}$$

where α and β are the direct and transverse optical stress factors.

The initial dielectric circle of an isotropic unloaded material

$$a^0_{xx}x^2 + a^0_{zz}z^2 - 1 = 0, \tag{3}$$

where $a^0_{xx} \equiv a^0_{zz} \equiv 1/\varepsilon$, goes over into an ellipse under load

$$a_{xx}x^2 + a_{zx}zx + a_{zz}z^2 - 1 = 0, \tag{4}$$

and when (2) is taken into account, becomes

$$\left(\frac{1}{\varepsilon} + \beta p_{zz}\right)x^2 + \left(\frac{1}{\varepsilon} + \alpha p_{zz}\right)z^2 + \frac{(\alpha-\beta)}{2} p_{zx}zx - 1 = 0. \tag{5}$$

This is the equation of an ellipse rotated through a certain angle ψ relative to the coordinate axes. This angle of rotation governs the influence of the tangential stresses on the dielectric properties of the material along the z axis.

If we use the notation $1/\varepsilon + \alpha p_{zz} = a_1^2$ and $1/\varepsilon + \beta p_{zz} = a_2^2$, then (5) becomes

$$\frac{x^2}{a_1^2} + \frac{z^2}{a_2^2} + \frac{2(\alpha-\beta)}{4a_1^2a_2^2} p_{zx}zx - \frac{1}{a_1^2a_2^2} = 0, \tag{6}$$

which may also be expressed in terms of the vibration equations

$$x = a_1\cos(\tau + \delta_1), \quad z = a_2\cos(\tau + \delta_2),$$

if we use the notation $(\alpha-\pi)/4a_1a_2p_{zx} = \cos\delta$ and $1/a_1^2a_2^2 = \sin^2\delta$. Here δ denotes the difference in the initial phases $(\delta_1 - \delta_2)$.

To go over to the intrinsic coordinates (ξ, η), where the ellipse is described by the equations

$$\xi = a\cos(\tau + \delta_0), \quad \eta = b\cos\left(\tau + \delta_0 + \frac{\pi}{2}\right)$$

(a and b are the principal semiaxes of the ellipse), the coordinate axes must be rotated through the angle ψ defined in terms of δ, a_1, and a_2 by means of the formula [6]

$$\tan 2\psi = \frac{2a_1a_2\cos\delta}{a_1^2 - a_2^2}.$$

TABLE 1. Changes in the Location of the Interference Fringes As a Function of the Applied Moment of the Forces (in fractions of the "critical" moment)

N	M						
	0	0,6	1,3	2,5	6,5	13	25
20	1,657	1,655	1,656	1,662	1,663	1,655	1,657
40	3,338	3,335	3,338	3,340	3,338	3,339	3,331
60	5,039	5,035	5,041	5,044	5,036	5,040	5,033
80	6,721	6,719	6,719	6,716	6,720	6,715	6,718
100	8,385	8,379	8,383	8,381	8,381	8,378	8,380
120	10,095	10,090	10,094	10,089	10,090	10,086	10,084
140	11,791	11,783	11,789	11,787	11,792	11,790	11,792
160	13,474	13,475	13,478	13,484	13,480	13,479	13,478
180	15,169	15,160	15,164	15,166	15,164	15,170	15,163
200	16,865	16,867	16,870	16,873	16,874	16,874	16,873
220	18,571	18,569	18,572	18,573	18,574	18,574	18,578
240	20,263	20,265	20,264	20,262	20,261	20,271	20,271
260	21,973	21,969	21,966	21,970	21,971	21,977	21,979
280	23,683	23,685	23,681	23,680	23,675	23,687	23,684
300	25,398	25,395	25,396	25,395	25,398	25,403	25,404
320	27,098	27,098	27,102	27,096	27,098	27,105	27,109
340	28,811	28,805	28,812	28,802	28,808	28,814	28,815
350	29,666	29,665	29,669	29,660	29,667	29,672	29,680

If the values of $\cos\delta$, a_1^2 and a_2^2 are substituted into this formula, we obtain the equality $\tan 2\psi = (1/2)p_{zx}/p_{zz}$ from which there follows that for equal tangential and normal stresses, the contribution of the tangential stresses to a change in the dielectric permittivity is about 3% of the contribution from the normal stress since the projection of the ellipse semiaxis changes so much for a rotation of $\simeq 13°$.

The computation performed was experimentally verified. For this the resin layer sensitive to the stresses was subjected for an invariant normal load to a tangential force analogous to that which the moment of the viscous forces from the fluid in a torsion flow in [1] exerts on this layer.

The total moment of the viscous forces acting when the tangential stresses reach the magnitude of the normal stresses ("critical" moment) was determined by the formula

$$M_0 = \int_0^R \int_0^{2\pi} \tau_{cr} r \cdot r dr d\varphi = \frac{\pi\eta\omega R^4}{2h},$$

where R is the radius of a disk; h, a gap; η, viscosity; and ω, angular velocity. Substitution of the numerical values from experiment [1] yields $M_0 = 3 \cdot 10^4$ dyne · cm.

The location of the interference fringes (in mm) on the length of the measuring disk radius is represented in the table for different torques. Here N is the fringe number and M is the moment of the applied forces in fractions of the "critical" moment of the viscous forces M_0.

The results of processing the experimental results are represented in Fig. 2. The moment of the forces M is plotted along the abscissa axis here, and the maximum fixed mutual shift in the interference fringes Δl along the ordinate axis.

It follows from a comparison between these data and the results of a shift in the interference fringes under the influence of normal stresses [1] that normal stresses 20-30 times less than the tangential are required to achieve the same shifts.

Therefore, the experiment confirms the computation and demonstrates the negligible contribution of tangential stresses in the shift of the interference fringes as compared to the normal stress under conditions of their equality.

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NONSTEADY FLOWS OF VISCOPLASTIC FLUIDS AT THE INITIAL SECTIONS OF PLANE CHANNELS

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The problem is formulated and the method of solving internal problems of rheodynamics of nonsteady flows of viscoplastic fluids is proposed.

Nonsteady motions of viscoplastic media are of considerable interest in connection with investigations of technological processes occurring under dynamic loading. A number of articles are devoted to an analysis of the rheodynamics of nonsteady flows on spatially steady sections, a review of which is given in [1]. The initial sections for steady flow of viscoplastic fluids were considered in [2-5]. Investigations of nonsteady flows at initial sections of channels have so far not been carried out.

We consider the flow of a fluid in a plane channel (Fig. 1). The velocity of the fluid at the inlet is constant over the cross section and equal to V. From physical considerations the entire flow region can be divided into two regions: a zone of shearing flow, adjoining the wall of the channel ($\delta < y \le h$), and a "quasisolid" core, where the velocity is constant ($0 \le y \le \delta$). We should note that the velocity of the quasisolid core U(x) varies along the channel axis. Under such conditions we can use the modified model of a viscoplastic fluid, which for $\tau \le \tau_0$ exhibits creep, i.e., slow flow with high viscosity. Then, the nonuniqueness of the velocity field in the transverse direction can be neglected; in this case, however, the model admits the dependence of U on the longitudinal coordinate.

The equations of motion in the boundary-layer approximation are

$$\begin{cases} \rho\left(\dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y}\right) = -\dfrac{\partial p}{\partial x} + \mu\dfrac{\partial^2 u}{\partial y^2}, & (\delta < y \leqslant h) \quad (1)\\ \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0, & (2) \end{cases}$$

$$\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x}\right) = -\frac{\partial p}{\partial x} - \frac{\tau_0}{\delta} \quad (0 \leqslant y \leqslant \delta) \tag{3}$$

(an investigation was carried out for the linear model of a viscoplastic Shvedov–Bingham medium).

Equation (3) for a quasisolid core contributes the term τ_0/δ. It indicates that the stresses on the boundary of the quasisolid core are equal to τ_0. In [2] for the steady-state problem in the zone of quasisolid motion, the Shiller approximation was assumed to be valid:

$$U\frac{\partial U}{dx} = -\frac{1}{\rho}\frac{dp}{dx}. \tag{4}$$

In our opinion, neglecting the term τ_0/δ can lead to sizable errors in the solution.

Let the medium be at rest for $t \le 0$, and for $t > 0$ let there be a flow with a constant flow rate. The initial and boundary conditions of the problem are the following:

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